A Context-Free Process as a Pushdown Automaton

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(joint work with Jos Baeten and Pieter Cuijpers)

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TCS seminar
Vrije Universiteit, Amsterdam / June 20, 2008

Project MoCAP

- Models of Computation: Automata and Processes

Automata + Interaction = Concurrency

Project MoCAP

- Models of Computation: Automata and Processes

  Automata + Interaction = Concurrency

- Separate development
- Integration
- Study similarities and differences

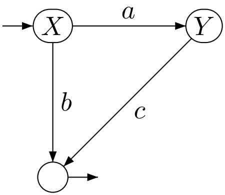
Right-linear grammar

Generates a regular language

$$X \longrightarrow aY \mid b$$
$$Y \longrightarrow c$$

(Finite) transition system

Accepts a regular language



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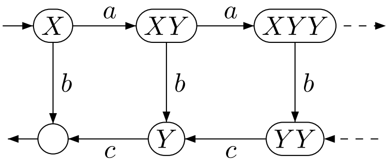

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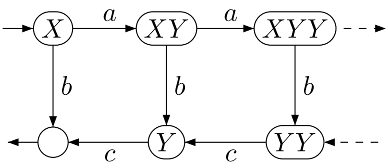

Theorem

For every context-free language there exists a pushdown automaton that accepts it.

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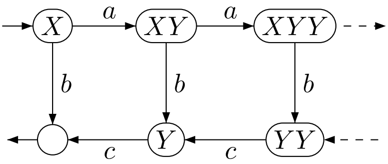
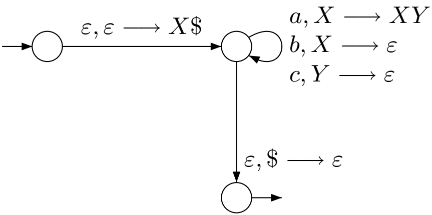
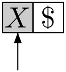


Pushdown automaton



Stack



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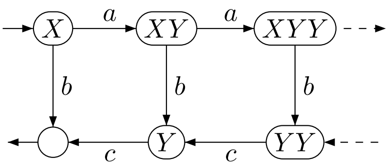

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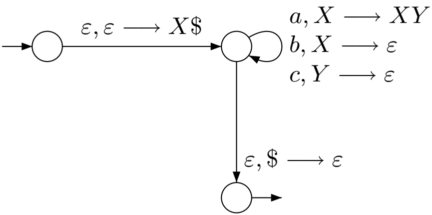

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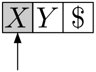

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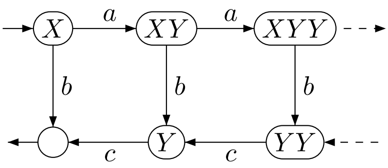

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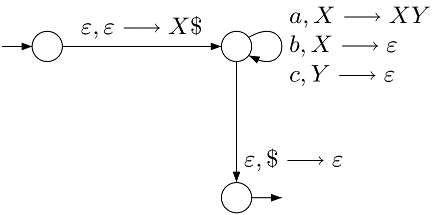

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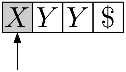

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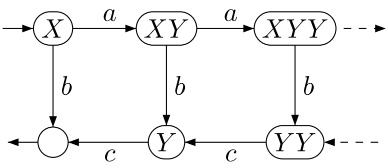

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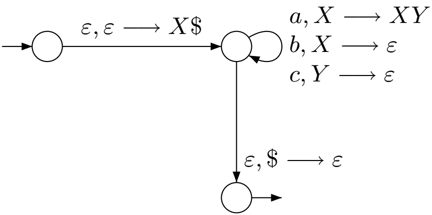

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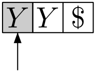

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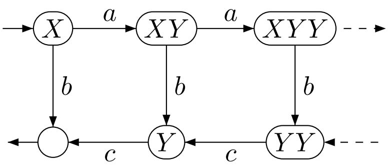

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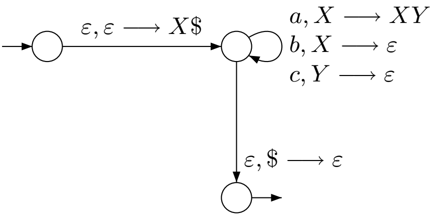

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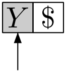

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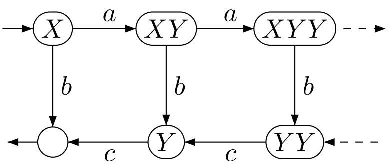

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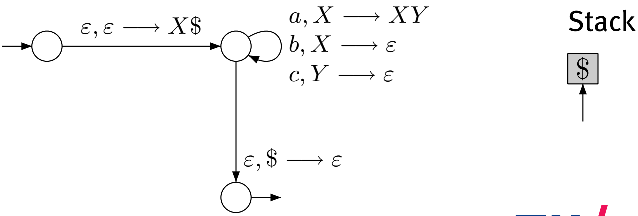

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Right-linear grammar

Generates a context-free language

$$
\begin{aligned}
X &\longrightarrow aXY \mid b \\
Y &\longrightarrow c
\end{aligned}
$$

Processes and Recursive Specifications

Right-linear grammar

Generates a context-free language

$$X \longrightarrow aXY \mid b$$
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Recursive specification over BPA

Specifies a context-free process

$$X = a \cdot (X \cdot Y) + b$$
$$Y = c$$

Restrict to:
finite and guarded specifications

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0 and 1

- Regular expressions use 0 (deadlock) and 1 (final state)
- Not done in grammars in automata theory
- Add it to recursive specifications

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- Regular expressions use $\mathbf{0}$ (deadlock) and $\mathbf{1}$ (final state)
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Main Theorem

Process theory enables us to...

- Model the data (a stack) as a process
- Make communication explicit
- Use bisimulation equivalences to preserve branching structure

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Theorem

Every context-free process is equivalent to a regular process communicating with a stack.

Specifications

Infinite recursive specification (infinite data set)

$$S_\varepsilon = \sum_{d\in D} ?d.S_d \qquad\qquad S_{d\sigma} = !d.S_\sigma + \sum_{e\in D} ?e.S_{ed\sigma}$$

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Finite recursive specification over BPA

$$S = T \cdot S \qquad T = \sum_{d \in D} ?d.T_d \qquad T_d = !d + T \cdot T_d$$

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Even smaller specification (over $\mathrm{BPA}_{\mathbf{0},\mathbf{1}}$)

$$S = \mathbf{1} + \sum_{d \in D} ?d.(S \cdot !d.S)$$

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Shorthand

$$\partial_{?,!}(P \parallel S \cdot !d_1.S \cdot \ldots \cdot !d_n.S) = P\langle d_1 \ldots d_n \rangle$$

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Context-free process

$$X = a.(X \cdot Y) + b.\mathbf{1}$$
$$Y = c.\mathbf{1}$$

Translated

$$\hat{X} = a.\mathrm{Push}(XY) + b.\mathrm{Push}(\mathbf{1})$$
$$\hat{Y} = c.\mathrm{Push}(\mathbf{1})$$

$$\mathrm{Push}(\mathbf{1}) = \mathrm{Ctrl}$$
$$\mathrm{Push}(\xi Y) = !Y.\mathrm{Push}(\xi)$$
$$\mathrm{Ctrl} = \sum_{V \in \mathcal{V}} ?V.\hat{V} + \mathbf{1}$$
$$S = \mathbf{1} + \sum_{V \in \mathcal{V}} ?V.S \cdot !V.S$$

Pushdown Automaton

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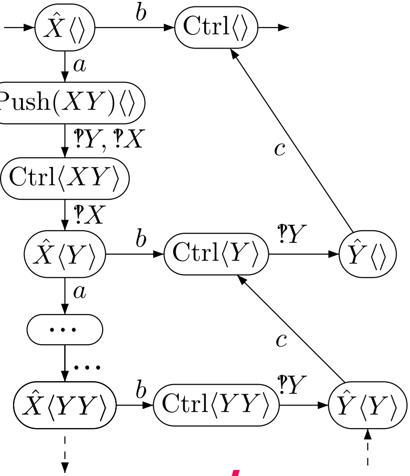
Transition system



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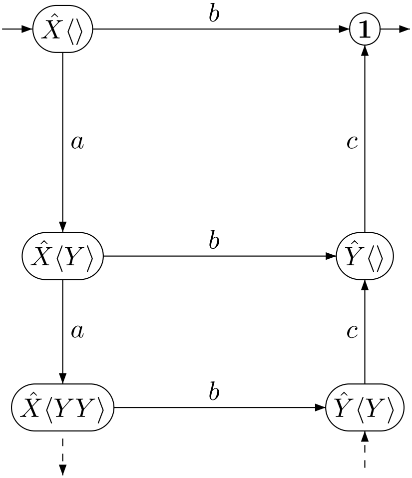
…modulo rooted br. bisim.



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Context-Free Processes with 1-summands

Context-free process

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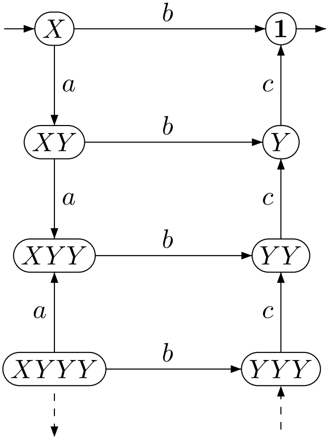
Transition system



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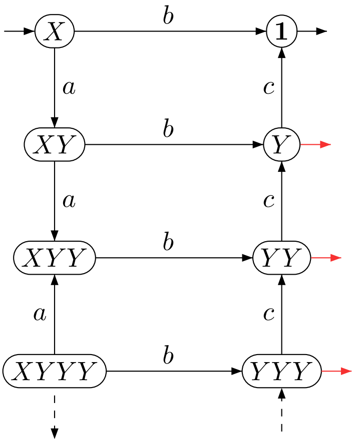

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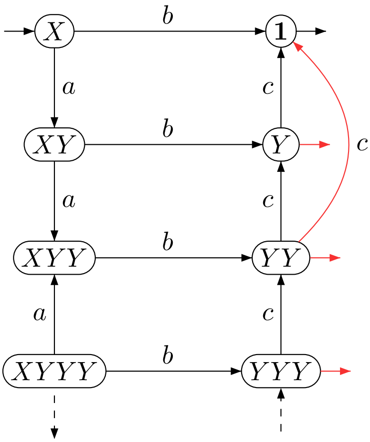

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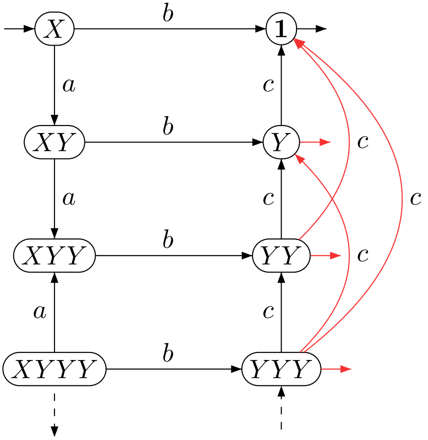

Context-Free Processes with 1-summands

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$$X = a.(X \cdot Y) + b.\mathbf{1},$$
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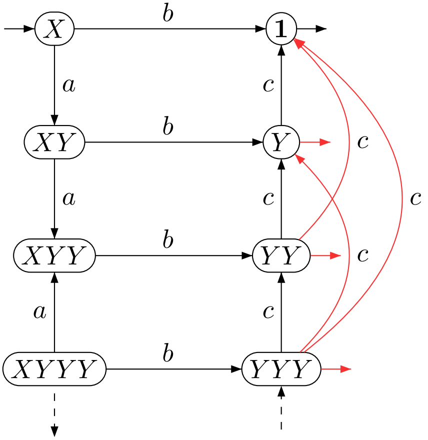
Translation adaptation

$$S = \mathbf{1} + \sum_{V \in \mathcal{V}} ?V.S \cdot !V.S$$

Transition system



Context-free process

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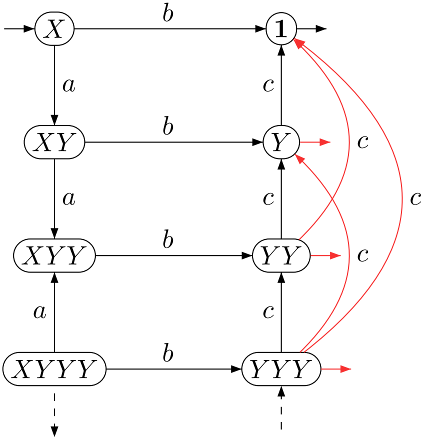
Translation adaptation

$$S = \mathbf{1} + \sum_{V \in \mathcal{V} - \mathcal{V}^{+1}} ?V.S \cdot !V.S$$
$$+ \sum_{V \in \mathcal{V}^{+1}} ?V.S \cdot (\mathbf{1} + !V.S)$$

for $\mathcal{V}^{+1} \subseteq \mathcal{V}$

Transition system



Context-Free Process Results

Unbounded branching

- Solution modulo contrasimulation
- Using partially forgetful stack, the prototypical context-free process

Without 1-summands

- Solution modulo rooted branching bisimulation
- Using normal stack, the prototypical context-free process (for BPA)

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Bounded branching

- Solution modulo rooted branching bisimulation!
- Using the partially forgetful stack

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Recursive specification over BPP

$$X = a.(X \parallel Y) + b.\mathbf{1}$$
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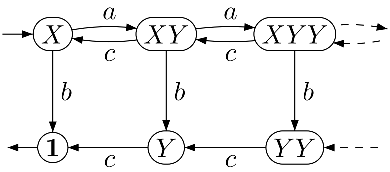
Transition system



Basic Parallel Processes

Recursive specification over BPP

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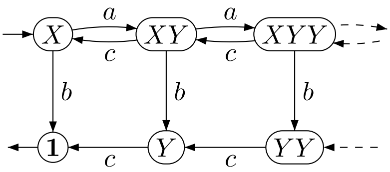
Transition system



The bag

$$B = \mathbf{1} + \sum_{d \in D} ?_i d.(B \parallel !_o d.\mathbf{1})$$

Basic Parallel Processes

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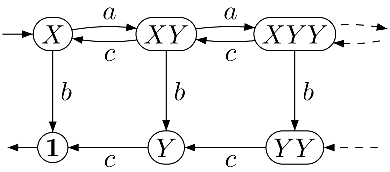

The bag

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Theorem

Every basic parallel process is equivalent to a regular process communicating with a bag.

Parallel Pushdown Automaton

Basic parallel process

$$X = a.(X \parallel Y) + b.\mathbf{1}$$
$$Y = c.\mathbf{1}$$

Translated

$$\hat{X} = a.\text{Push}(XY) + b.\text{Push}(\emptyset)$$
$$\hat{Y} = c.\text{Push}(\emptyset)$$

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Parallel Pushdown Automaton

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$$B = \mathbf{1} + \sum_{V \in \mathcal{V} - \mathcal{V}^{+\mathbf{1}}} ?_i V.(B \parallel !_o V.\mathbf{1}) + \sum_{V \in \mathcal{V}^{+\mathbf{1}}} ?_i V.(B \parallel (!_o V.\mathbf{1} + \mathbf{1})$$

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Basic Parallel Process Results

All cases

- Solution modulo rooted branching bisimulation
- Using partially forgetful bag, the prototypical basic parallel process

Remark

Using 1 made the solution easier

Concluding Remarks

Proved Theorems

- Made communication explicit
- Introduced 0 and 1, dealt with complications
- The (partially forgetful) stack is the prototypical context-free process
- The (partially forgetful) bag is the prototypical basic parallel process

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Future work

- Reverse case, maybe with 1?
- Queues?

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Questions?

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