Models of Computation: Automata and Processes

An Overview

Paul van Tilburg

(joint work with Jos Baeten, Bas Luttik and Pieter Cuijpers)

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University of Technology

Where innovation starts

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Introduction

Automata & Formal Language theory

- *Back in the days:* different model and real-world computers
- Fixed input string
- Input separated from output
- Batch process

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- Computers are reactive systems
- Interaction much more important

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- *Note:* Provides very useful models of computation

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Introduction (2)

Process theory

- Split off, separate development
- Focuses on interaction
- Deals with concurrent setting

Integration

- Attempt reveals differences and similarities
- Use *analogies* to make the integration explicit
- Increase understanding of both theories

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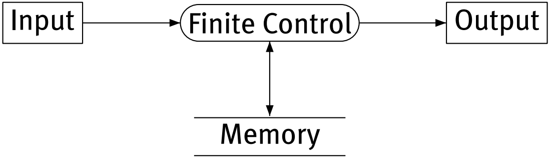
- *Practical side:* merge in undergraduate curriculum course

Turing machine



- Control is discrete: states and transitions: automaton
- Input, output: string or word over alphabet
- Alphabet: action, instruction, information

Finite Automaton

- Corresponds to regular language
- No memory!
- Two equivalences: language equivalence and isomorphism

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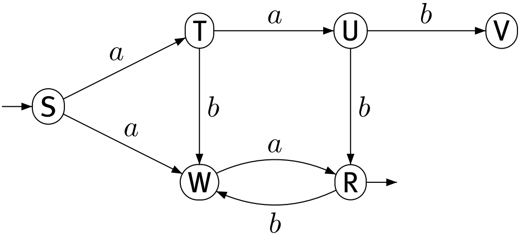
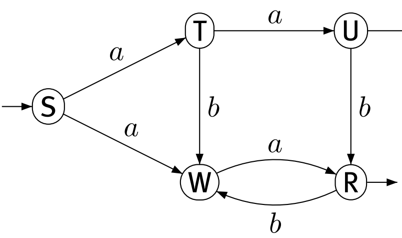
Grammars and Recursive Specifications





$$
\begin{aligned}
S &= aT + aW \\
T &= aU + bW \\
U &= bV + bR \\
V &= \mathbf{0} \\
W &= aR \\
R &= bW + \mathbf{1}
\end{aligned}
$$

- From Finite Automaton to recursive specification

From Recursive Specification to Automaton

$$\frac{}{\mathbf{1}\downarrow} \qquad \frac{}{ax \xrightarrow{a} x}$$

$$\frac{x \xrightarrow{a} x'}{x+y \xrightarrow{a} x'} \qquad \frac{y \xrightarrow{a} y'}{x+y \xrightarrow{a} y'} \qquad \frac{x\downarrow}{x+y\downarrow} \qquad \frac{y\downarrow}{x+y\downarrow}$$

$$\frac{t \xrightarrow{a} x \quad P = t}{P \xrightarrow{a} x} \qquad \frac{t\downarrow \quad P = t}{P\downarrow}$$

- Structural Operational Semantics [Plotkin, *JLAP*, 2004]

Similarities with Process Algebra

- Finite Automaton = finite labelled transition system
- Grammar = recursive specification over $\mathbf{0}, \mathbf{1}, +, \cdot, a$
- Regular expression = closed term over $\mathbf{0}, \mathbf{1}, +, \cdot, a, {}^*$

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Basic Process Algebra

- $\mathbf{0}$ inaction, unsuccessful termination, deadlock
- $\mathbf{1}$ empty process, skip, successful termination
- $a\_$ action prefix
- $\_ + \_$ alternative composition, choice
- $\_ \cdot \_$ sequential composition

[Baeten, Basten, Reniers, *Process Algebra*, Cambridge UP, 2009]

Bisimulation

- In process theory a difference equivalent is used
- Expose interaction and preserve choices

Definition

We call the largest symmetric relation R such that

- if $p \xrightarrow{a} p'$ then there exists q' such that $q \xrightarrow{a} q'$ and $p' \; R \; q'$
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- if $p\downarrow$ implies $q\downarrow$ and vice versa

a *bisimulation* relation

- If $(p, q) \in R$, then p and q are *bisimilar* (notation: $p \leftrightarrows q$)

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A *regular language* is a language equivalence class of a finite (non-deterministic) automaton

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- A regular process is given by a recursive specification over the signature $\mathbf{0}, \mathbf{1}, a, +$
- Processes given by deterministic automata, and by regular expressions, form a subclass
  [Baeten, Corradini, Grabmayer, *JACM* 2007]

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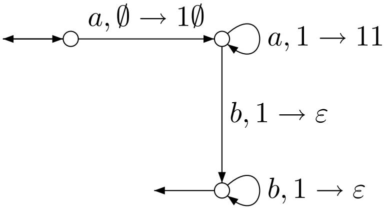
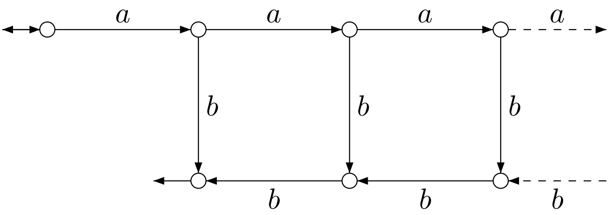
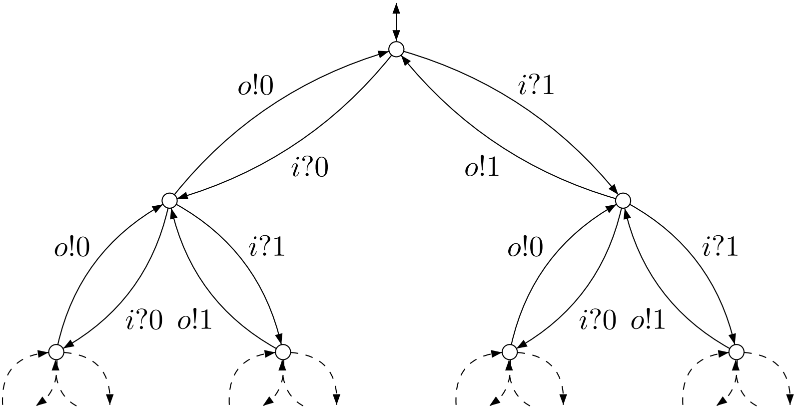
Pushdown Automaton







$$S = \mathbf{1} + \sum_{d \in \mathcal{D}} i?d.S \cdot o!d.S$$

Pushdown Process

Theorem

A process p is a pushdown process iff there is a regular process q with

$$p \leftrightarrows_{b} \tau_{i,o}(\partial_{i,o}(q \parallel S))$$

- Proof in [Baeten, Cuijpers, Luttik, van Tilburg, *FSEN*, 2009]

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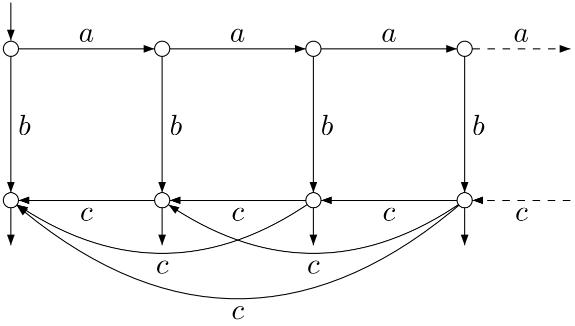
Recursive specification

Every recursive specification over $\mathrm{BPA}_{\mathbf{0},\mathbf{1}}$ with bounded branching denotes a pushdown process

- Example: $X = \mathbf{1} + aX \cdot b\mathbf{1}$

$$X \xrightarrow{a} X \cdot b\mathbf{1} \xrightarrow{a} X \cdot b\mathbf{1} \cdot b\mathbf{1} \ldots$$

Problem with 1-summands



$$X = aX \cdot Y + b\mathbf{1}$$
$$Y = \mathbf{1} + c\mathbf{1}$$

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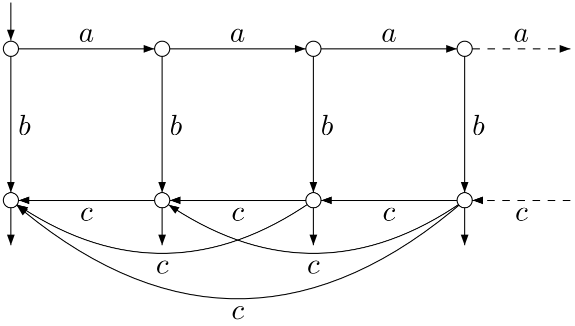

$$X = aX \cdot Y + b\mathbf{1}$$
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- Recursive specifications over $\text{BPA}_{\mathbf{0},\mathbf{1}}$ can lead to unboundedness
- Cannot be done by our pushdown process due to stack and finite control
- Can be solved using a *forgetful stack* [Baeten, Cuijpers, van Tilburg, *CONCUR*, 2008]

What About Context-Free Processes?

- Context-free languages correspond to language accepted by PDAs
- Not the case with bisimulation! [Moller, 1996]
- *Fix:* do not allow for *pop choice* (to ensure existence specification)

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A process is a pop choice-free pushdown process iff it is definable by a transparency-restricted recursive specification [FSEN, 2009]

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- Not every pushdown process is context-free
- Decidability of bisimulation shown for this class!

Definition

A parallel pushdown automaton gives a parallel pushdown process

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Basic Parallel Process

Definition

A parallel pushdown automaton gives a parallel pushdown process

Theorem

A process p is parallel pushdown iff there is a regular process q with

$$p \leftrightarroweq_b \tau_{i,o}(\partial_{i,o}(q \parallel B))$$

where B is the bag: $B = \mathbf{1} + \sum_{d \in \mathcal{D}} i?d.(B \parallel o!d.\mathbf{1})$

[Baeten, Cuijpers, van Tilburg, EXPRESS, 2008]

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Definition

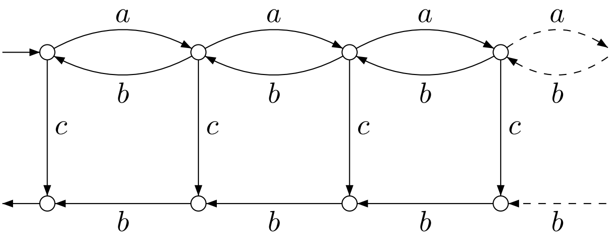
A *basic parallel process* is given by a guarded recursive specification over the signature $\mathbf{0}, \mathbf{1}, +, a\_, \parallel$

- Any basic parallel process is a parallel pushdown process

Example

$$X = c.\mathbf{1} + a.(X \parallel b.\mathbf{1})$$

is basic parallel, parallel pushdown and pushdown but not context-free



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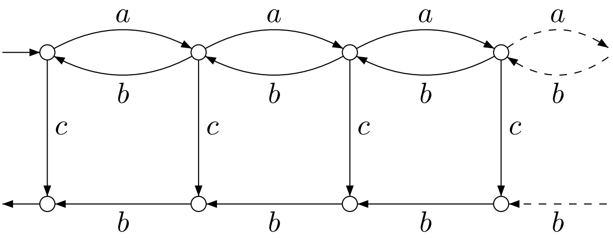

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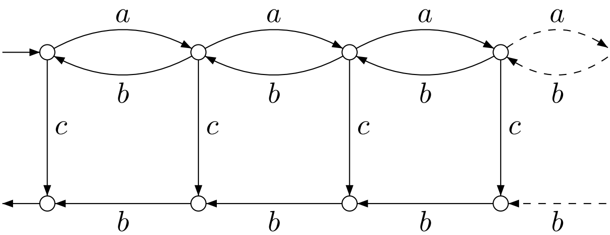


The bag is basic parallel, parallel pushdown but not pushdown, nor context-free

The stack is context-free, pushdown but not basic parallel, nor parallel pushdown

Computable Process

Definition

A computable process is a bisimulation equivalence class of a computable transition system

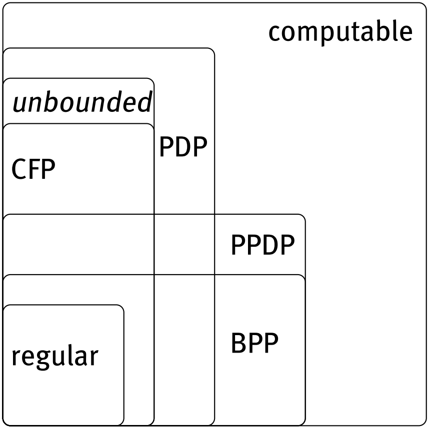
Theorem

A process is computable iff it is an abstraction of a process given by a guarded recursive specification over communication algebra [FSEN, 2009]

Theorem

A process is computable iff it can be written as a regular process communicating with two stacks [FSEN, 2009]

Process Classes



Conclusion

- Integration of automata theory and process theory is beneficial for both theories
- This integrated theory can be a first-year course in any academic bachelor program in computer science (or related subjects)
- Draft syllabus available

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Thank you!

Questions?