Decidability of Bisimulation for Sequential and Basic Parallel Processes with 0 and 1

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Where innovation starts

Decidability of bisimilarity

Given a process theory is there an algorithm that for every two processes in the theory that can determine whether they are bisimilar or not

- Decidability results important for verification
- Proof is trivial for finite state transition system
- It gets interesting for infinite systems

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History

Sequential processes

- "Decidability of Bisimulation Equivalence for Process Generating Context-Free Languages" [Bergstra, Baeten & Klop 1987]
  - Result for normed BPA $(a, +, \cdot)$
- Several simplified/different versions appeared [Caucal 1986, Groote 1992, Hüttel & Stirling 1991]
- Later Caucal's proof was extended to all of BPA [Christensen, Hüttel & Stirling 1995]

Parallel processes

- Meanwhile proof given for all of BPP $(a, +, \|)$ [Christensen, Hüttel & Moller 1993]

Deadlock and the Empty Process

Why do we need $\mathbf{0}$ (deadlock) and $\mathbf{1}$ (empty process)?

- Faithful translation of context-free grammars
  - $\mathbf{0}$ for missing productions
  - $\mathbf{1}$ for empty productions

$$X \longrightarrow aXY \mid bZ \qquad\qquad X = aXY + b\mathbf{0}$$

$$Y \longrightarrow c \mid \varepsilon \qquad\qquad Y = c + \mathbf{1}$$

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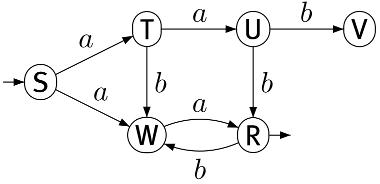
- Represent finite automata
  - $\mathbf{0}$ to represent a state without outgoing transitions
  - $\mathbf{1}$ to represent (intermediate) termination in a certain state



$$S = a.T + a.W \qquad V = \mathbf{0}$$
$$T = a.U + b.W \qquad W = a.R$$
$$U = b.R + b.V \qquad R = b.W + \mathbf{1}$$

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Deadlock and the Empty Process (2)

- Another more complicated example:

$$X = aXY + b$$
$$Y = c + \mathbf{1}$$

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Proof sketch for BPA with $\mathbf{0}$

- Caucal/Bosscher extended proof for BPA with $\mathbf{0}$ [Srba 2001, Bosscher 1997]
- Reused the decidability result for BPA by Christensen, Hüttel & Stirling

Proof sketch

- Reduce a $\mathrm{BPA}_{\mathbf{0}}$ specification to BPA such that it preserves bisimilarity

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Proof sketch

- Reduce a $\mathrm{BPA}_{\mathbf{0}}$ specification to BPA such that it preserves bisimilarity
- Introduce a fresh variable $D = dD$ to act as deadlock

$$X = aXX + b + d\mathbf{0}$$

$$\hat{X} = a\hat{X}\hat{X} + b + dD$$
$$D = dD$$

- Since decidability for BPA is known and reduction is bisimilarity preserving, decidability for $\mathrm{BPA}_{\mathbf{0}}$ is proved

Proof suggestion for BPA with $\mathbf{0}$ and $\mathbf{1}$

Could we do a similar reduction to $\mathrm{BPA}_{\mathbf{0},\mathbf{1}}$?

Proof suggestion

- Reduce a $\mathrm{BPA}_{\mathbf{0},\mathbf{1}}$ specification to $\mathrm{BPA}_{\mathbf{0}}$ such that is preserves bisimilarity
- Introduce a fresh action $\surd$ to replace $\mathbf{1}$-summands

$$X = ab\mathbf{1} + b\mathbf{1} \qquad \hat{X} = ab + b$$

$$Y = a\mathbf{1} + \mathbf{1} \qquad \hat{Y} = a + \surd$$

$$Z = b\mathbf{1} \qquad \hat{Z} = b$$

- It is obvious that: $ab\mathbf{1} + b\mathbf{1} = X \leftrightarrow YZ = (a\mathbf{1} + \mathbf{1})b\mathbf{1}$
- But: $ab + b = \hat{X} \not\leftrightarrow \hat{Y}\hat{Z} = (a + \surd)b$
- So the the reduction does not preserve bisimilarity

Fixing the proof

Consider sequential processes:

$$X_1 \cdot X_2 \cdot \ldots \cdot X_{n-1} \cdot X_n$$

Fixing the proof

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Definition (Transparency-restricted)

A sequence of variables is *transparency-restricted* if in all sequences of variables reachable from it only the last variable may be transparent

- This subclass of sequential processes is non-trivial
  - It can describe the finite automata
  - The Stack process is a member of this class:

$$S = \mathbf{1} + \sum_{d \in D} ?d.T_d S$$

$$T_d = !d.\mathbf{1} + \sum_{e \in D} ?e.T_e T_d$$

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Fixing the proof (2)

- Using *transparency-restricted* sequential processes we have no more intermediate $\surd$-actions, they only occur at the end.
- Our previous example ($X \leftrightarroweq YZ$) no longer causes trouble:

$$X = ab\mathbf{1} + b\mathbf{1} \qquad Y = a\mathbf{1} + \mathbf{1} \qquad Z = b\mathbf{1}$$

  because YZ is not transparency restricted
- Another example:

$$X = X_1 \cdot X_2 \cdot \ldots \cdot X_n \overset{?}{\leftrightarroweq} Y_1 \cdot Y_2 \cdot \ldots \cdot Y_m = Y$$

Proof problems for BPA with $\mathbf{0}$

Why not adapt the original proof by Christensen, Hüttel & Stirling?

- Generate a bisimulation relation from a finite bisimulation basis
- The basis contains pairs of bisimilar sequences of variables that can be seen as rules
- Two kind of pairs:
  1. $(X, Y_1 Y_2 \ldots Y_n)$ for each X
     - No longer finite!
     - Consider: $X = a.X + \mathbf{1}$
     - $(X \leftrightarrows X^k)$ for any k

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     - $(X \leftrightarroweq X^k)$ for any k
  2. $(X_1X_2\ldots X_k, Y_1Y_2\ldots Y_l)$ as indecomposable pairs
     - Also not longer finite!
- For the proof to work one needs to be able to check whether the set of pairs is a basis
- However, the basis is no longer finite

Why the extension of BPP with 0 and 1 is easier

- $\mathrm{BPA}_{\mathbf{0},\mathbf{1}}$ processes:

$$X_1 \cdot X_2 \cdot \ldots \cdot X_n \xrightarrow{a} X_i' \cdot \ldots \cdot X_n$$

- $\mathrm{BPP}_{\mathbf{0},\mathbf{1}}$ processes:

$$Y_1 \parallel Y_2 \parallel \ldots \parallel Y_n \xrightarrow{a} Y_1 \parallel Y_2 \parallel \ldots \parallel Y_j' \parallel \ldots \parallel Y_n$$

When adding 0 and 1...

- Parallel processes gain deadlock and impure termination
- Sequential processes gain deadlock and impure termination, but also forgetfulness and unbounded branching
- Situation for $\mathrm{BPP}_{\mathbf{0},\mathbf{1}}$ much simpler; the reduction approach works

Concluding Remarks

Results

- Decidability for transparency-restricted $\mathrm{BPA}_{\mathbf{0,1}}$
  - Captures finite automata
  - Closer to faithful translation of context-free grammars
- Decidability for $\mathrm{BPP}_{\mathbf{0,1}}$

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Results

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Future work

- Decidability for whole of $\mathrm{BPA}_{\mathbf{0},\mathbf{1}}$
- Decidability for PA

- Technical report out soon!

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Questions?

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