Finite Equational Bases for Fragments of CCS with Restriction and Relabelling

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(joint work with Luca Aceto[2], Anna Ingólfsdóttir[2], Bas Luttik[1])

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Outline

Introduction & Preliminaries

Restriction

Relabelling

Combinations

Concluding Remarks

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Introduction

- Based on earlier work:
  - A Finite Equational Base for CCS with Left Merge and Communication Merge – Luca Aceto, Wan Fokkink, Anna Ingólfsdóttir, Bas Luttik (2006)
  - Finite Equational Bases for CCS with Restriction – Master's Thesis – Paul van Tilburg (2007)
- CS-Report 08-08 contains details and proofs
- Goal: show you how these proofs work

Process Algebras and Equations

Process algebra:

- set of elements (processes)
- operations defined on this set

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Process equation:

- pair of process terms: $p \approx q$
- valid iff $[\![p]\!]_* = [\![q]\!]_*$ for all variable substitutions $*$

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- operations defined on this set

Process equation:

- pair of process terms: $p \approx q$
- valid iff $[\![p]\!]_* = [\![q]\!]_*$ for all variable substitutions $*$

Equational theory: set of *all* valid equations

Equational base: set of valid equations from which all other valid equations can be derived

CCS: Calculus of Communication Systems – Robin Milner

Syntax: set of process terms $\mathcal{T}$ generated by

$$\mathsf{T} ::= \mathbf{0} \mid x \mid a.\mathsf{T} \mid \mathsf{T} + \mathsf{T} \mid T \parallel T \mid T \setminus L \mid T[f]$$
$$(a \in \mathcal{A}, x \in \mathcal{V}, L \subset \mathcal{A}, f : \mathcal{A} \to \mathcal{A})$$

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Previous result: finite equational base for a fragment of CCS

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$$(a \in \mathcal{A}, x \in \mathcal{V}, L \subset \mathcal{A}, f : \mathcal{A} \to \mathcal{A})$$

Previous result: finite equational base for a fragment of CCS

Goal: finite equational base for full CCS

Result: finite equational base for CCS without communication

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BCCS

BCCS: basic fragment of CCS

Syntax: set of process terms $\mathcal{T}$ generated by

$$\mathsf{T} ::= \mathbf{0} \mid x \mid a.\mathsf{T} \mid \mathsf{T} + \mathsf{T} \qquad (a \in \mathcal{A}, x \in \mathcal{V})$$

set of closed process terms $\mathcal{T}^C$: terms without variables

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set of closed process terms $\mathcal{T}^C$: terms without variables

Semantics: labelled transition system for a term $p \in \mathcal{T}$ given by

$$1\ \frac{}{a.p \xrightarrow{a} p} \qquad 2\ \frac{p \xrightarrow{a} p'}{p + q \xrightarrow{a} p'} \qquad 3\ \frac{q \xrightarrow{a} q'}{p + q \xrightarrow{a} q'}$$

BCCS

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Bisimulation: largest symmetric relation $\leftrightarrows$ such that

if $p \xrightarrow{a} p'$ and $p \leftrightarrows q$, then $\exists q'$ s.t. $q \xrightarrow{a} q'$ and $p' \leftrightarrows q'$

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BCCS (2)

Construct the process algebra $\mathbf{P}$:

Elements: $\leftrightarroweq$ is an equivalence relation:

$\mathcal{T}/\leftrightarroweq$ results in classes $[p] = \{q \mid p \leftrightarroweq q\}$

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BCCS (2)

Construct the process algebra $\mathbf{P}$:

Elements: $\leftrightarroweq$ is an equivalence relation:

$\mathcal{T}/\leftrightarroweq$ results in classes $[p] = \{q \mid p \leftrightarroweq q\}$

Operators: $\leftrightarroweq$ is a congruence:

if $p \leftrightarroweq p'$, then $a.p \leftrightarroweq a.p'$ and
if $p \leftrightarroweq p'$ and $q \leftrightarroweq q'$, then $p + q \leftrightarroweq p' + q'$

induces operations on the equivalence classes

$$\mathbf{0} = [\mathbf{0}], \qquad a.[p] = [a.p], \qquad [p] + [q] = [p + q]$$

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BCCS (3)

$\mathbf{P}$ has a well-known equational base $\mathcal{E}$:

$$
\begin{array}{lll}
\text{(A1)} & x + y & \approx y + x \\
\text{(A2)} & (x + y) + z & \approx x + (y + z) \\
\text{(A3)} & x + x & \approx x \\
\text{(A4)} & x + \mathbf{0} & \approx x
\end{array}
$$

Theorem

$\mathcal{E}$ is a (finite) equational base for $\mathbf{P}$

Proof Strategy

Soundness: if $p \approx q$ derivable, then $p \leftrightarroweq q$

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Completeness: if $p \leftrightarroweq q$, then $p \approx q$ derivable

Proof Strategy

Soundness: if $p \approx q$ derivable, then $p \leftrightarrows q$

Completeness: if $p \leftrightarrows q$, then $p \approx q$ derivable

How to prove?

- find normal forms s and t such that $p \approx s$ and $q \approx t$
- $s \leftrightarrows t$ means that $[\![s]\!]_\nu = [\![t]\!]_\nu$ for all $\nu : \mathcal{V} \to \mathbf{P}$
- find a distinguishing valuation $* : \mathcal{V} \to \mathbf{P}$ for s, t s.t.
  if $s \not\approx t$ then $[\![s]\!]_* \neq [\![t]\!]_*$ for all s, t

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Proof Strategy

Soundness: if $p \approx q$ derivable, then $p \leftrightarroweq q$

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How to prove?

- find normal forms s and t such that $p \approx s$ and $q \approx t$
  - so: if $s \leftrightarroweq t$, then $s \approx t$ derivable
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Normal Forms

For every process p there exists a normal form s such that

$$s = \sum_{i \in I} a_i.s_i + \sum_{j \in J} x_j$$

Branching degree:

the number of outgoing transitions of a process

Example

- the branching degree of $a.\mathbf{0} + a.\mathbf{0} + b.\mathbf{0} + b.c.\mathbf{0}$ is 3
- the branching degree of

$$\xi_i = \sum_{a \in \mathcal{A}} \sum_{j=1}^{i} a^j.\mathbf{0} = a.\mathbf{0} + a.a.\mathbf{0} + \cdots + a^i.\mathbf{0} + b.\mathbf{0} + \ldots \text{ is } i \cdot |\mathcal{A}|$$

Completeness Proof

Distinguishing valuation

Let $w \geq 1$ and let $\lceil\cdot\rceil : \mathcal{V} \to (\mathbb{N} - \{0, 1\})$ be some injective function

$$\diamond_w(x) = a.\psi_{\lceil x \rceil \cdot w} \text{ with } \psi_i = \sum_{j=1}^{i} a^i.\mathbf{0}$$

Completeness Proof

Distinguishing valuation

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Example

Distinguishing $a.s$ from x:

$$[\![a.s]\!]_{\diamond_w} \xrightarrow{a} s$$

$$[\![x]\!]_{\diamond_w} = a.\psi_{\lceil x \rceil \cdot w} \xrightarrow{a} \psi_{\lceil x \rceil \cdot w} \quad \text{(branching degree is } \lceil x \rceil \cdot w\text{)}$$

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Adding Restriction

$\mathbf{P}_{\setminus}$: BCCS extended with restriction

Syntax: set of process terms $\mathcal{T}_{\setminus}$ generated by

$$\mathsf{T} ::= \ \ldots \ | \ \mathsf{T} \setminus L \qquad (L \subseteq \mathcal{A})$$

Adding Restriction

$\mathbf{P}_\backslash$: BCCS extended with restriction

Syntax: set of process terms $\mathcal{T}_\backslash$ generated by

$$\mathsf{T} ::= \ \ldots \ | \ \mathsf{T} \setminus L \qquad (L \subseteq \mathcal{A})$$

Semantics: labelled transition system for a term $p \in \mathcal{T}$ given by

$$4\ \frac{p \xrightarrow{a} p' \qquad a, \overline{a} \notin L}{p \setminus L \xrightarrow{a} p' \setminus L}$$

Example

if $p = (a.\mathbf{0} + b.\mathbf{0}) \setminus \{a\}$, then $p \stackrel{a}{\not\longrightarrow}$, but $p \xrightarrow{b} \mathbf{0}$.

Equational Base

$\mathbf{P}_{\setminus}$ has an equational base $\mathcal{E}_{\setminus}$:

$$
\begin{array}{lll}
\text{(RS1a)} & x \setminus \emptyset & \approx x \\
\text{(RS1b)} & x \setminus \mathcal{A} & \approx \mathbf{0} \\
\text{(RS2)} & \mathbf{0} \setminus L & \approx \mathbf{0} \\
\text{(RS3)} & a.x \setminus L & \approx \begin{cases} \mathbf{0} & \text{if } a, \overline{a} \in L \\ a.(x \setminus L) & \text{if } a, \overline{a} \notin L \end{cases} \\
\text{(RS4)} & (x + y) \setminus L & \approx x \setminus L + y \setminus L \\
\text{(RS6)} & (x \setminus L) \setminus K & \approx x \setminus (L \cup K)
\end{array}
$$

Distinguishing Issues

Normal Forms

For every process p there exists a normal form s such that

$$s = \sum_{i \in I} a_i.s_i + \sum_{j \in J} x_j \setminus L_j \qquad (L_j \subset \mathcal{A})$$

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Example

Distinguish issues given $\mathcal{A} = \{a, b\}$, $\mathcal{V} = \{x, y\}$:

- $a.s$ from $x \setminus \{b\}$
- $x \setminus L$ from $y \setminus L$
- $x \setminus \emptyset$ from $x \setminus \{a\} + x \setminus \{b\}$

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Completeness Proof

Distinguishing valuation

Let $w \geq 1$ and let $\lceil \cdot \rceil : \mathcal{V} \rightarrow (\mathbb{N} - \{0, 1\})$ be some injective function

$$\diamond_w(x) = \sum_{a \in \mathcal{A}} a.\xi_{\lceil x \rceil \cdot w} \text{ with } \xi_i = \sum_{a \in \mathcal{A}} \sum_{j=1}^{i} a^i.\mathbf{0}$$

Completeness Proof

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Example

Distinguishing $a.s$ from $x \setminus \{b\}$:

$$[\![a.s]\!]_{\diamond_w} \xrightarrow{a} s$$

$$[\![x \setminus \{b\}]\!]_{\diamond_w} \xrightarrow{a} \xi_{\lceil x \rceil \cdot w} \setminus \{b\}$$ (branching degree is $\lceil x \rceil \cdot w \cdot |\mathcal{A} - \{b\}|$)

Completeness Proof

Distinguishing valuation

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$$\diamond_w(x) = \sum_{a \in \mathcal{A}} a.\xi_{\lceil x \rceil \cdot w} \text{ with } \xi_i = \sum_{a \in \mathcal{A}} \sum_{j=1}^{i} a^i.\mathbf{0}$$

Example

Distinguishing $x \setminus L$ from $y \setminus L$:

$$[\![x \setminus L]\!]_{\diamond_w} \xrightarrow{a} \xi_{\lceil x \rceil \cdot w} \setminus L \qquad (a \notin L, \text{br.deg. } \lceil x \rceil \cdot w \cdot |\mathcal{A} - L|)$$

$$[\![y \setminus L]\!]_{\diamond_w} \xrightarrow{a} \xi_{\lceil y \rceil \cdot w} \setminus L \qquad (a \notin L, \text{br.deg. } \lceil y \rceil \cdot w \cdot |\mathcal{A} - L|)$$

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Completeness Proof

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$$\diamond_w(x) = \sum_{a \in \mathcal{A}} a.\xi_{\lceil x \rceil \cdot w} \text{ with } \xi_i = \sum_{a \in \mathcal{A}} \sum_{j=1}^{i} a^i.\mathbf{0}$$

Example

Distinguishing $x \setminus \emptyset$ from $x \setminus \{a\} + x \setminus \{b\}$:

$$[\![x \setminus \emptyset]\!]_{\diamond_w} \xrightarrow{a} \xi_{\lceil x \rceil \cdot w}$$

$$[\![x \setminus \{a\} + x \setminus \{b\}]\!]_{\diamond_w} \xrightarrow{a} \xi_{\lceil y \rceil \cdot w} \setminus \{b\} \text{ or}$$

$$\xrightarrow{b} \xi_{\lceil y \rceil \cdot w} \setminus \{a\}$$

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Adding Relabelling

$\mathbf{P}_{[]}$: BCCS extended with relabelling

Syntax: set of process terms $\mathcal{T}_{[]}$ generated by

$$\mathsf{T} ::= \ldots \mid \mathsf{T}[f] \qquad (f : \mathcal{A} \to \mathcal{A})$$

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Syntax: set of process terms $\mathcal{T}_{[]}$ generated by

$$\mathsf{T} ::= \ldots \mid \mathsf{T}[f] \qquad (f : \mathcal{A} \to \mathcal{A})$$

Semantics: labelled transition system for a term $p \in \mathcal{T}$ given by

$$5\ \frac{p \xrightarrow{a} p'}{p[f] \xrightarrow{f(a)} p'[f]}$$

Example

if $p = (a.\mathbf{0} + b.c.\mathbf{0})[b \mapsto a]$, then $p \xrightarrow{a} \mathbf{0}$ and $p \xrightarrow{a} c.\mathbf{0}$.

Equational Base

$\mathbf{P}_{[]}$ has an equational base $\mathcal{E}_{[]}$:

$$
\begin{array}{lll}
\text{(RL1)} & x[Id] & \approx x \\
\text{(RL2)} & \mathbf{0}[f] & \approx \mathbf{0} \\
\text{(RL3)} & (a.x)[f] & \approx f(a).(x[f]) \\
\text{(RL4)} & (x+y)[f] & \approx x[f] + y[f] \\
 & & \\
\text{(RL6)} & (x[f])[g] & \approx x[g \circ f]
\end{array}
$$

Distinguishing Issues

Normal Forms

For every process p there exists a normal form s such that

$$s = \sum_{i \in I} a_i.s_i + \sum_{j \in J} x_j[f_j] \qquad (f_j : \mathcal{A} \to \mathcal{A})$$

Distinguishing Issues

Normal Forms

For every process p there exists a normal form s such that

$$s = \sum_{i \in I} a_i.s_i + \sum_{j \in J} x_j[f_j] \qquad (f_j : \mathcal{A} \to \mathcal{A})$$

Example

Distinguish issues given $\mathcal{A} = \{a, b\}$, $\mathcal{V} = \{x, y\}$:

- $a.s$ from $x[b \mapsto a]$
- $x[Id]$ from $x[a \mapsto b, b \mapsto a]$

Distinguishing Issues

Normal Forms

For every process p there exists a normal form s such that

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- $a.s$ from $x[b \mapsto a]$
- $x[Id]$ from $x[a \mapsto b, b \mapsto a]$

Completeness Proof

Distinguishing valuation

Let $\lfloor\cdot\rfloor : \mathcal{A} \to \mathbb{P}$ be some injective function,
let $w \in \mathbb{P}$ larger than any number in the range of $\lfloor\cdot\rfloor$, and
let $\lceil\cdot\rceil : \mathcal{V} \to \{m \in \mathbb{P} \mid m > w\}$ be another injective function

$$\diamond_w(x) = a.\zeta_{\lceil x\rceil,w} \text{ with } \zeta_{i,w} = a.\mathbf{0} + \sum_{b\in\mathcal{A}} \sum_{j=1}^{w} b^{i\cdot\lfloor b\rfloor^j}.\mathbf{0}$$

Distinguishing valuation

Let $\lfloor\cdot\rfloor : \mathcal{A} \to \mathbb{P}$ be some injective function,
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Example

Distinguishing $a.s$ from $x[b \mapsto a]$:

$$[\![a.s]\!]_{\diamond_w} \xrightarrow{a} s$$

$$[\![x[b \mapsto a]]\!]_{\diamond_w} \xrightarrow{a} \zeta_{\lceil x\rceil,w}[b \mapsto a] \qquad \text{(branching degree } 1 + w\cdot|\mathcal{A}|)$$

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Completeness Proof

Distinguishing valuation

Let $\lfloor\cdot\rfloor : \mathcal{A} \to \mathbb{P}$ be some injective function,
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Example

Distinguishing $x[Id]$ from $x[a \mapsto b, b \mapsto a]$:

$$[\![x[Id]]\!]_{\diamond_w} \xrightarrow{a} \zeta_{\lceil x\rceil,w}$$

$$[\![x[a \mapsto b, b \mapsto a]]\!]_{\diamond_w} \xrightarrow{b} \zeta_{\lceil y\rceil\cdot w}[a \mapsto b, b \mapsto a]$$

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$\mathbf{P}_{\setminus,[]}$ has an equational base $\mathcal{E}_{\setminus,[]}$ combining $\mathcal{E}_{\setminus}$, $\mathcal{E}_{[]}$, and:

(RR1) $x[f] \setminus L \approx (x \setminus f^{-1}(L))[f]$

(RR2) $(x \setminus L)[f] \approx (x \setminus L)[g]$ if $f \restriction (\mathcal{A} - L) = g \restriction (\mathcal{A} - L)$

Normal Forms

For every process p there exists a normal form s such that

$$s = \sum_{i \in I} a_i.s_i + \sum_{j \in J} (x_j \setminus L_j)[f_j] \qquad (L_j \subset \mathcal{A}, f_j : \mathcal{A} \to \mathcal{A})$$

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Distinguishing valuation

Let $\lfloor\cdot\rfloor : \mathcal{A} \to \mathbb{P}$ be some injective function,
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let $\lceil\cdot\rceil : \mathcal{V} \to \{m \in \mathbb{P} \mid m > w\}$ be another injective function

$$\diamond_w(x) = \sum_{a\in\mathcal{A}} a.\chi_{\lceil x\rceil,w} \text{ with } \chi_{i,w} = \sum_{a\in\mathcal{A}} \left(a.\mathbf{0} + \sum_{j=1}^{w} a^{i\cdot\lfloor a\rfloor^j}.\mathbf{0} \right)$$

Adding Interleaving

Syntax

set of process terms $\mathcal{T}^{\parallel}$ generated by

$$\mathsf{T} ::= \ldots \mid \mathsf{T} \parallel \mathsf{T} \mid \mathsf{T} \mathbin{\|\!\_} \mathsf{T}$$

Standard axioms

$$\begin{array}{lll}
\text{(LM1)} & x \mathbin{\|\!_} \mathbf{0} & \approx x \\
\text{(LM2)} & \mathbf{0} \mathbin{\|\!_} x & \approx \mathbf{0} \\
\text{(LM3)} & a.x \mathbin{\|\!_} y & \approx a.(x \parallel y) \\
\text{(LM4)} & (x + y) \mathbin{\|\!_} z & \approx x \mathbin{\|\!_} z + y \mathbin{\|\!_} z \\
\text{(LM5)} & (x \mathbin{\|\!_} y) \mathbin{\|\!_} z & \approx x \mathbin{\|\!_} (y \parallel z) \\
 & & \\
\text{(M)} & x \parallel y & \approx x \mathbin{\|\!_} y + y \mathbin{\|\!_} x
\end{array}$$

Adding Interleaving

Syntax

set of process terms $\mathcal{T}^{\parallel}$ generated by

$$\mathsf{T} ::= \ldots \mid \mathsf{T} \parallel \mathsf{T} \mid \mathsf{T} \mathbin{⫝} \mathsf{T}$$

Distributive axioms

Due to absence of communication:

$$\text{(RS5)} \quad (x \mathbin{⫝} y) \setminus L \approx x \setminus L \mathbin{⫝} y \setminus L$$

$$\text{(RL5)} \quad (x \mathbin{⫝} y)[f] \approx x[f] \mathbin{⫝} y[f]$$

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Adding Interleaving (2)

Normal Forms

For every process p there exists a normal form s such that

$$s = \sum_{i \in I} a_i . s_i + \sum_{j \in J} (x_j \setminus L_j)[f_j] \mathbin{|\!\lfloor} s_j \qquad (L_j \subset \mathcal{A}, f_j : \mathcal{A} \to \mathcal{A})$$

Previously given proofs still work!

Example

$$[\![x \setminus L \mathbin{|\!\lfloor} s]\!]_{\diamond w} \xrightarrow{a} (\xi_{\lceil x \rceil \cdot w} \setminus L) \parallel s$$

Concluding Remarks

Results

- Proved completeness of finite equational bases for fragments
  - with restriction
  - with relabelling
  - with combination of restriction and relabelling
  - with and without interleaving
- While recursion has been left out, the addition changes nothing

Future work

- Non-trivial addition of communication merge remains!

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Questions?